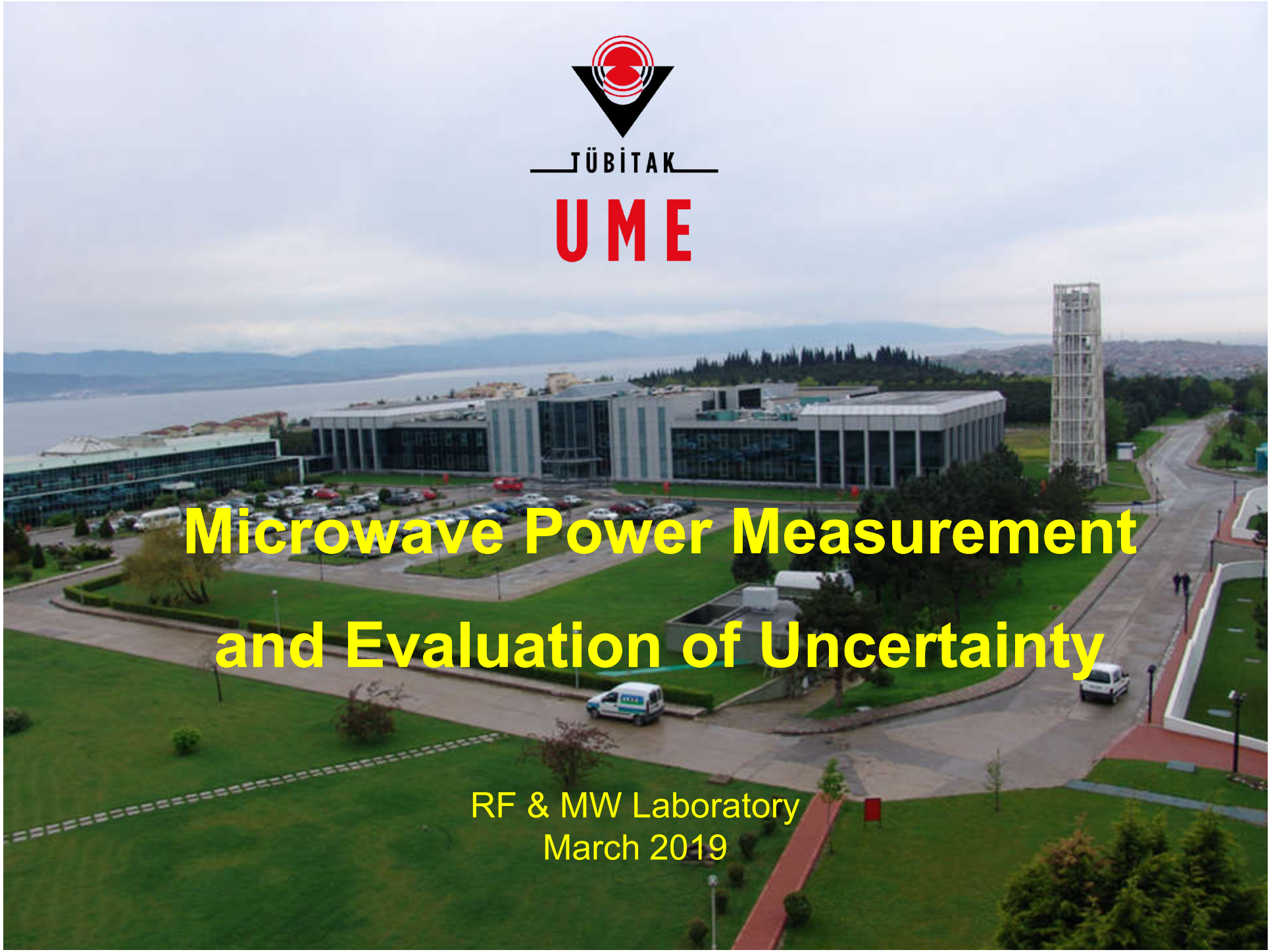


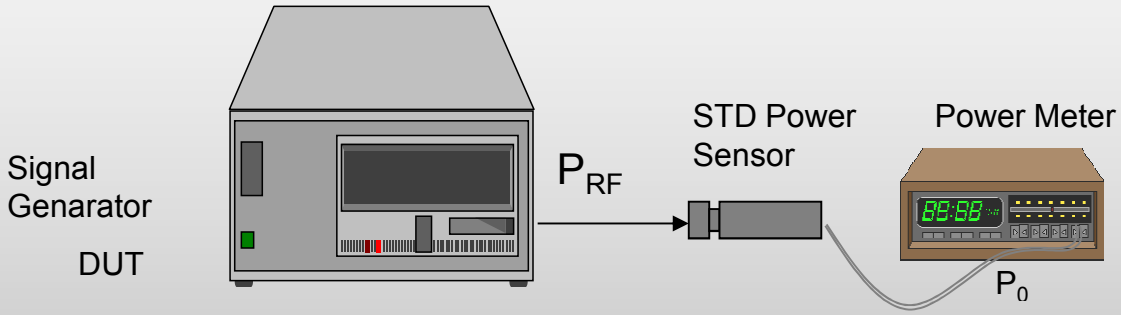
TÜBİTAK

UME

Microwave Power Measurement and Evaluation of Uncertainty

RF & MW Laboratory
March 2019

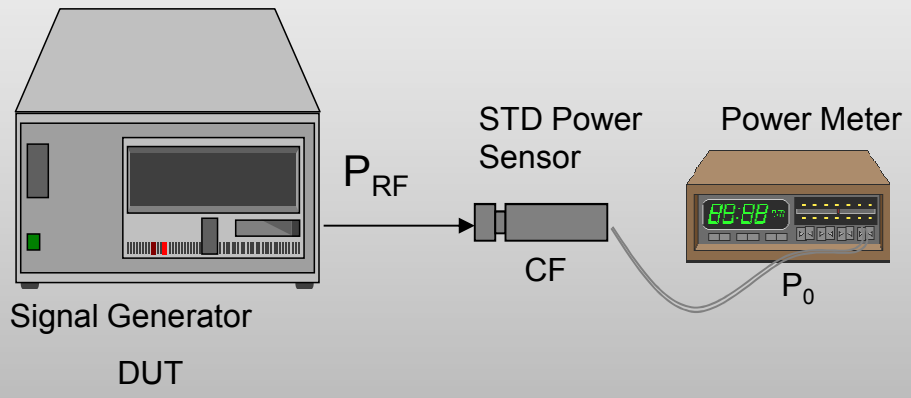
Example: Set up of power measurment



$$P_{RF} = \frac{P_0}{CF_{STD}} M_S$$

Steps for uncertainty evaluation

1. Model function:



Model Function $$\left(\frac{P_0 + \delta Pm}{CF_{STD}}\right) \cdot Ms$$

NOTE:

This is an example of model function.It is valid for nominal 1 mW power value. It will be different (including different parameters)for the other power levels.

Model function is determined according to the measurement setup used

Steps for uncertainty evaluation

2. Determine the uncertainty components and their uncertainties :
   - **Partial uncertainty calculation** of the components
   - Calculation of the **sensitivity coefficients(c_i)** of components
   - Calculation of the **partial variance** of components

$$\left(\frac{\partial f}{\partial x_i}\right)^2 \cdot u^2(x_i)$$

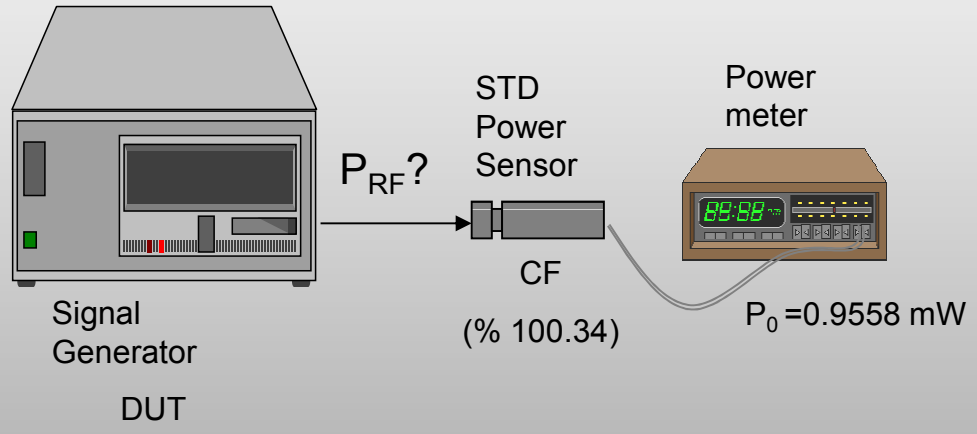
3. Evaluation of the combined uncertainty

$$U_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 \cdot u^2(x_i)}$$

4. Evaluation of the expanded uncertainty

$$U_y = k \cdot u_c(y)$$

Example: Power measurment and its uncertainty evaluation



Uncertainty Budget

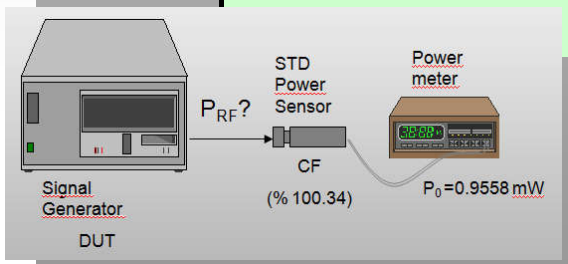
Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,76E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2

Measured value	P_{RF}	0,000952761	W		
Values given at the certificate	P_{RF}	9,528E-01	±	0,0118	mW

Total Variance	u_{PRF}^2 =	3,49E-11	W^2
Standard Uncertainty	u_{PRF} =	5,90E-06	W
Expanded Uncertainty	U_{PRF} =	1,18E-05	W
(k=2, Confidence Level: 95%)			
Relative uncertainty	U_{PRFrel} =	12,39	mW/W



Uncertainty Budget

Model Function:

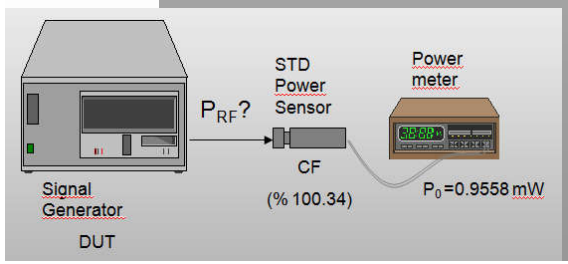
$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	[illegible],10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,72[illegible]4E-1[illegible]	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,7[illegible]E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,6[illegible]E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,[illegible]9E-12	W^2
										Total Variance		u_{PRF}^2 =	3[illegible]9E-11	W^2
Measured value		P_{RF}	0,000952761	W						Combined Uncertainty		u_{PRF} =	5[illegible]90E-06	W
Values given at the certificate		P_{RF}	9,528E-01	±	[illegible],0118	mW				Expanded Uncertainty		U_{PRF} =	[illegible],18E-05	W
										(k=2, Confidence Level: 95%)				
										Relative uncertainty		U_{PRFrel} =	12,39	mW/W

$$\frac{Exp.Std.Dev.}{\sqrt{n}}$$

$$\frac{\partial P_{RF}}{\partial P_0} = \frac{Ms}{CF_{STD}}$$

$$u_{P_0}^2 \cdot 1^2 \cdot c_{P_0}^2$$



Belirsizlik Bütçesi - örnek

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

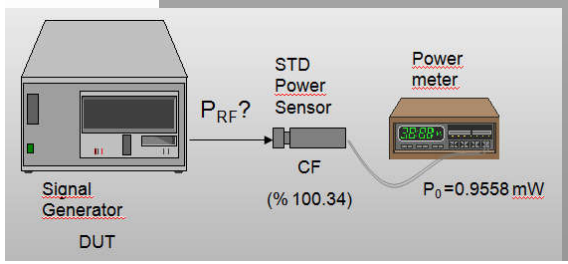
Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	22E-06	-	U-Type	0,7	c_{Ms}	00095	W	6,6E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	58E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_a}	1,0E+00	-	39E-12	W^2
										Total Variance		u_{PRF}^2 =	49E-11	W^2
Measured value		P_{RF}	0,000952761	W						Combined Uncertainty		u_{PRF} =	90E-06	W
Values given at the certificate		P_{RF}	9,528E-01	±	0118		mW			Expanded Uncertainty		U_{PRF} =	18E-05	W
										(k=2, Confidence Level: 95%)				
										Relative uncertainty		U_{PRFrel} =	12,39	mW/W

Uncertainty taken from certificate

Convert to k=1 level uncertainty value is divided into 2 (or multiplied by 0.5)

$$u_{CF_{STD}}^{\ 2} \cdot 0{,}5^2 \cdot C_{CF_{STD}}^{\ 2}$$

$$\frac{\partial P_{RF}}{\partial CF_{STD}} = \frac{-(P_0 + \delta Pm_acc + \delta Pm_rfpow) \cdot Ms}{CF_{STD}^{\ 2}}$$

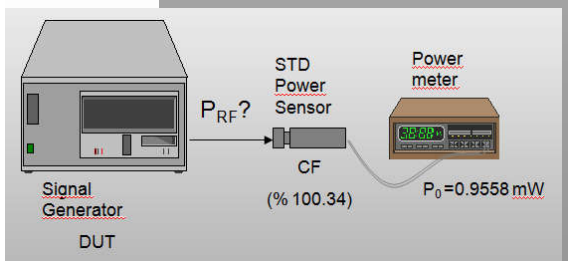


Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,76E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2
Measured value		P_{RF}	0,000952761	W						Total Variance		u_{PRF}^2 =	3,49E-11	W^2
Values given at the certificate		P_{RF}	9,528E-01	±	0,0118	mW				Combined Uncertainty		u_{PRF} =	5,90E-06	W
										Expanded Uncertainty		U_{PRF} =	1,18E-05	W
										(k=2, Confidence Level: 95%)				
										Relative uncertainty		U_{PRFrel} =	1,239	mW/W



$$2 \cdot \rho_{source} \cdot \rho_{Sensor}$$

Due to the prob. Distribution

$$\frac{\partial P_{RF}}{\partial Ms} = \frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}}$$

$$u_{Ms}^2 \cdot 0{,}7^2 \cdot c_{Ms}^2$$

Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

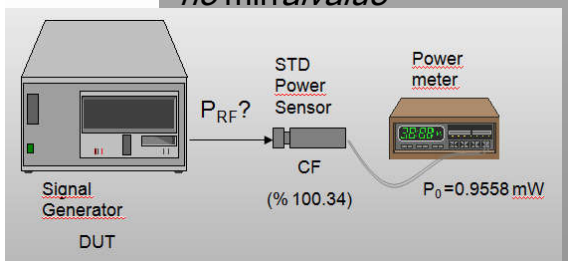
Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,76E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,8[illegible]E-12	W^2
Measured value		P_{RF}	0,000952761	W										
Values given at the certificate		P_{RF}	9,528E-01	±	0,[illegible]118		mW							
										Total Variance		u_{PRF}^2 =	3,[illegible]9E-11	W^2
										Combined Uncertainty		u_{PRF} =	5,[illegible]0E-06	W
										Expanded Uncertainty		U_{PRF} =	1[illegible]8E-05	W
										(k=2, Confidence Level: 95%)				
										Relative uncertainty		U_{PRFrel} =	12,39	mW/W

$$\frac{Uncer.refpower}{nominalvalue}.reading = \frac{0.005\,mW}{0.999\,mW}.9.56E-4W$$

Due to the prob. Distribution

$$u_{P_{m_rfpow}}^{2} \cdot 0.5^2 \cdot c_{P_{m_rfpow}}^{2}$$

$$\frac{\partial P_{RF}}{\partial Pm_rfpow} = \frac{Ms}{CF_{STD}}$$



Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,76E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2

Measured value	P_{RF}	0,000952761	W		
Values given at the certificate	P_{RF}	9,528E-01	±	0,0118	mW

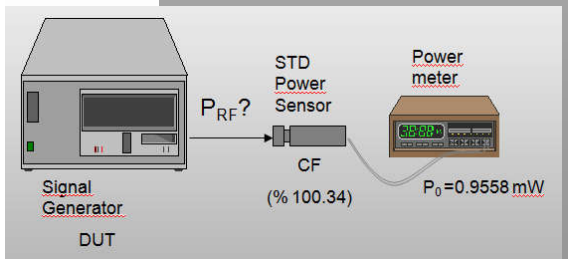
Total Variance	u_{PRF}^2 =	3,49E-11	W^2
Combined Uncertainty	u_{PRF} =	5,9[illegible]E-06	W
Expanded Uncertainty	U_{PRF} =	1,1[illegible]E-05	W
(k=2, Confidence Level: 95%)			
Relative uncertainty	U_{PRFrel} =	2,39	mW/W

$$\frac{Acc_powermeter(0.5\%)}{2}.reading = \frac{0.005}{2}.9.56E-4W$$

Due to the prob. Distribution

$$u_{Pm_acc}^{\ 2} \cdot 0.6^2 \cdot c_{Pm_acc}^{\ 2}$$

$$\frac{\partial P_{RF}}{\partial Pm_acc} = \frac{Ms}{CF_{STD}}$$



Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
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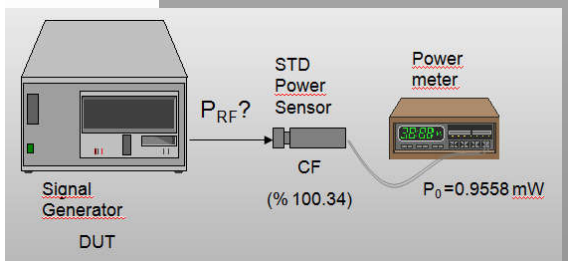
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2
Measured value		P_{RF}	0,000952761	W										
Values given at the certificate		P_{RF}	9,528E-01	±	0,0118	mW								

Total Variance	u_{PRF}^2 =	3,49E-11	W^2
Combined Uncertainty	u_{PRF} =	5,90E-06	W
Expanded Uncertainty	U_{PRF} =	1,18E-05	W
(k=2, Confidence Level: 95%)			
Relative uncertainty	U_{PRFrel} =	12,39	mW/W



$$u_{P_0}^2 + u_{CF_{STD}}^2 + u_{Ms}^2 + u_{\delta Pm_rfpow}^2 + u_{\delta Pm_acc}^2$$

Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
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Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
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Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2
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Values given at the certificate		P_{RF}	9,528E-01	±	0,0118	mW								

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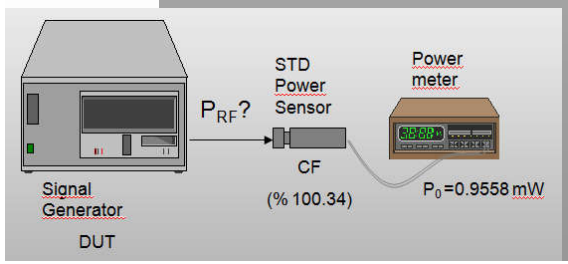
Combined Uncertainty	u_{PRF} =	5,90E-06	W
Expanded Uncertainty	U_{PRF} =	1,18E-05	W
(k=2, Confidence Level: 95%)			
Relative uncertainty	U_{PRFrel} =	12,39	mW/W

$$\sqrt{u_{P_0}^2 + u_{CF_{STD}}^2 + u_{Ms}^2 + u_{\delta Pm_rfpow}^2 + u_{\delta Pm_acc}^2}$$



Uncertainty budget

Model Function:

$$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF_{STD}} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
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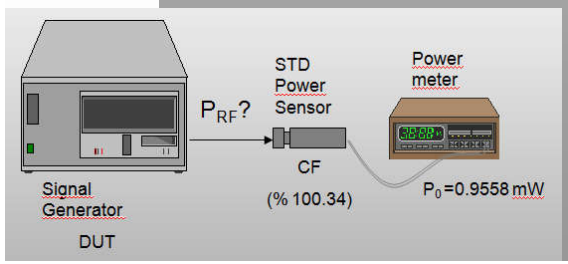
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2

Measured value	P_{RF}	0,000952761	W		
Values given at the certificate	P_{RF}	9,528E-01	±	0,0118	mW

Total Variance	u_{PRF}^2 =	3,49E-11	W^2
Combined Uncertainty	u_{PRF} =	5,90E-06	W
Expanded Uncertainty	U_{PRF} =	1,18E-05	W
(k=2, Confidence Level: 95%)			
Relative Uncertainty	U_{PRFrel} =	12,39	mW/W

$$2 \cdot \sqrt{u_{P_0}^2 + u_{CF_{STD}}^2 + u_{Ms}^2 + u_{\delta Pm_rfpow}^2 + u_{\delta Pm_acc}^2}$$



Uncertainty budget

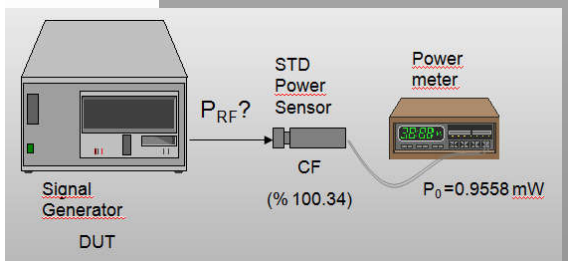
Model Function: $$P_{RF} = \left(\frac{P_0 + \delta Pm_acc + \delta Pm_rfpow}{CF} \right) \cdot Ms$$

Uncertainty Components					Partial Uncertainty					Sensivity Coefficients			Partial Variance	
Description		Symbol	Estimate value	Unit	Symbol	Value	Unit	Prob. Disrt.	Multiplier	Symbol	Value	Unit	Value	Unit
power reading with power meter	Type A	P_0	9,56E-04	W	u_{P0}	8,80E-08	W	Normal	1	c_{P0}	0,9966	-	7,6916E-15	W^2
Uncertainty of calibration factor of standard power sensor	Type B	CF_{STD}	1,0034	-	u_{CFSTD}	1,10E-02	-	Normal, k=2	0,5	c_{CFSTD}	-0,0009	W	2,7274E-11	W^2
Empedance mismatch	Type B	M_S	1,0000	-	u_{Ms}	1,22E-06	-	U-Type	0,7	c_{Ms}	0,00095	W	6,76E-19	W^2
Uncertainty due to the rf power of power meter	TypeB	Pm_rfpow	0			4,78E-06	W	Normal, k=2	0,5	c_{Pm_rfpow}	1,0E+00	-	5,68E-12	W^2
Uncertainty due to the power meter accuracy	Type B	Pm_acc	0	-	u_{Pm}	2,39E-06	W	Rectangular	0,6	c_{Pm_acc}	1,0E+00	-	1,89E-12	W^2

Measured value	P_{RF}	0,000952761	W		
Values given at the certificate	P_{RF}	9,528E-01	±	0,0118	mW

Total Variance	u_{PRF}^2 =	3,49E-11	W^2
Combined Uncertainty	u_{PRF} =	5,90E-06	W
Expanded Uncertainty	U_{PRF} =	1,18E-05	W
(k=2, Confidence Level: 95%)			
Relative uncertainty	U_{PRFrel} =	12,39	mW/W

$$\frac{ExpandedUncertainty\ (mW)}{P_{RF}\ (W)}$$



$$P(dBm) = 10 \cdot \log \frac{P_{RF}}{1mW}$$

Power defined in logaritmic

$$U(P_{dBm}) = 2 \cdot \sqrt{\left(\frac{\partial P_{dBm}}{\partial P_{RF}}\right)^2 \cdot \left(u_{P_{RF}}\right)^2}$$

Power uncertainty in logaritmic

$$c_{PRF} = \frac{\partial P_{dBm}}{\partial P_{RF}} = \frac{10}{P_{RF} \cdot \ln(10)}$$

Sensivity coefficient of power

$$U(P_{dBm}) = 2 \cdot \sqrt{(4543.94)^2 \cdot \left(6.49 \cdot 10^{-6}\right)^2} = 0.059\, dB$$