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Evaluation of Measurement Uncertainty and Example of Power Measurement

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Gebze – KOCAELİ

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  RF Power Measurement

What is Uncertainty ?

What does external world know about uncertainty ?

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Uncertainty

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Dictionary

Search for a word

uncertainty

/ʌnˈsəːt(ə)nti/

noun

the state of being uncertain.
"times of uncertainty and danger"
synonyms: unpredictability, unreliability, riskiness, chanciness, precariousness, unsureness
More

- something that is uncertain or that causes one to feel uncertain.
  plural noun: uncertainties
  "financial uncertainties"
  synonyms: doubt, qualm, misgiving, apprehension, quandary, dilemma, reservation, niggl
  scruple, second thought, query, question, question mark, suspicion
  "she pushed the anxious uncertainties out of her mind"

Translations, word origin, and more definitions

Fee

Uncertainty - Wikipedia
https://en.wikipedia.org/wiki/Uncertainty
Uncertainty refers to epistemic situations involving imperfect or unknown information. It applie
predictions of future events, to physical measurements that are ...
Measurement uncertainty · Uncertainty principle · Volatility, uncertainty ...

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Looks impressive !

Uncertainty - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Uncertainty**
Uncertainty is a term used in subtly different ways in a number of fields, including philosophy, physics, statistics, economics, finance, insurance, psychology, ...
Uncertainty principle - Uncertainty (film) - Measurement uncertainty

Uncertainty principle - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Uncertainty**_principle
Historically, the uncertainty principle has been confused with a somewhat ...

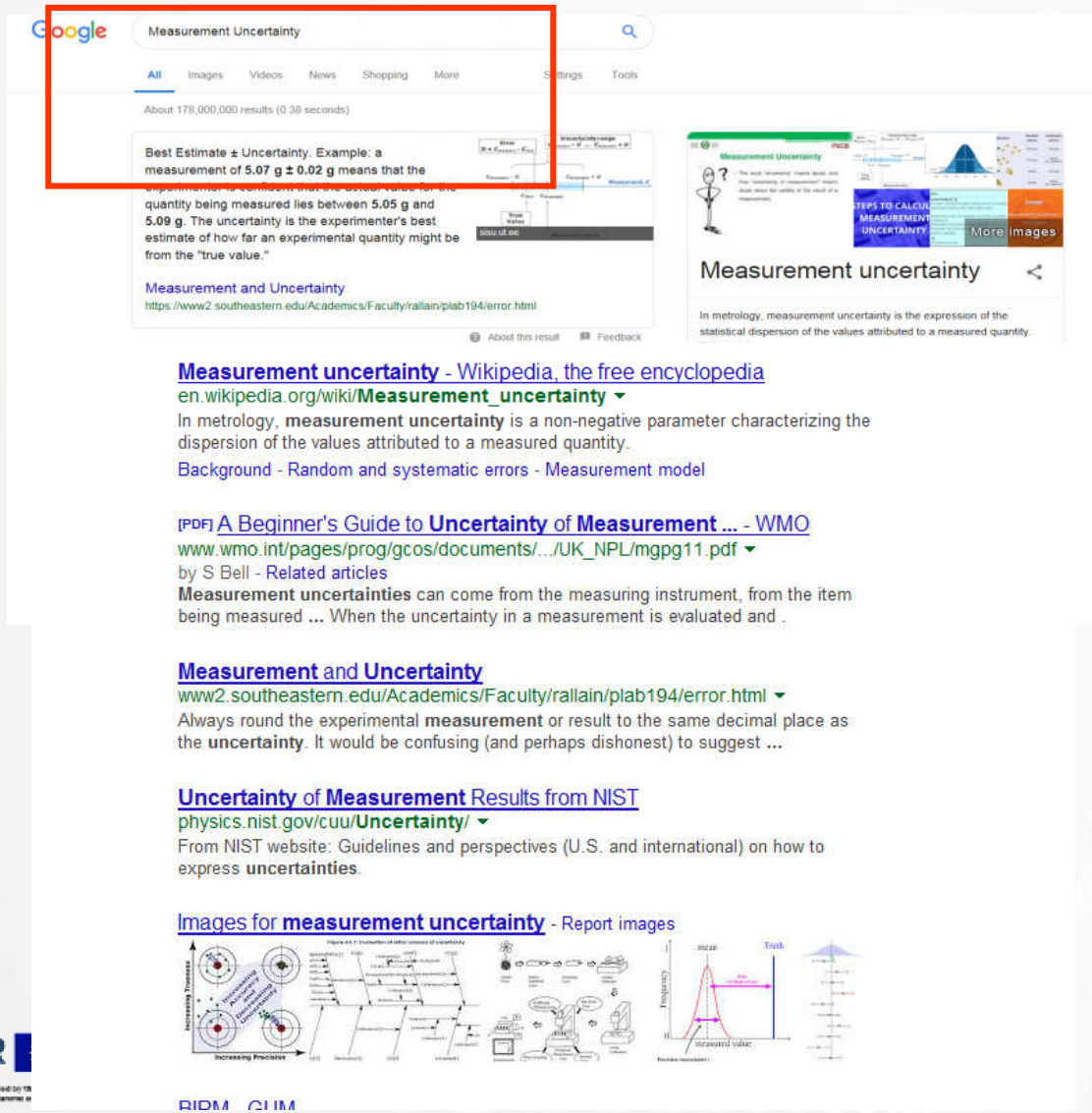
Uncertainty (2009) - IMDb
www.imdb.com/title/tt1086216/
★★★☆☆ Rating: 5.8/10 - 4,774 votes
Directed by Scott McGehee, David Siegel. With Lynn Collins, Joseph Gordon- Levitt, Assumpta Serna, Louis Arcella. A young couple, in love and facing a ...
User Reviews - Parents Guide - Full Cast & Crew - Soundtracks

Understanding **Uncertainty**
understanding**uncertainty**.org/
Welcome to the site that tries to make sense of chance, risk, luck, **uncertainty** and probability. Mathematics won't tell us what to do, but we think that ...

Uncertainty - Rotten Tomatoes
www.rottentomatoes.com/m/**uncertainty**/

What is Uncertainty ?

What does external world know about uncertainty ?



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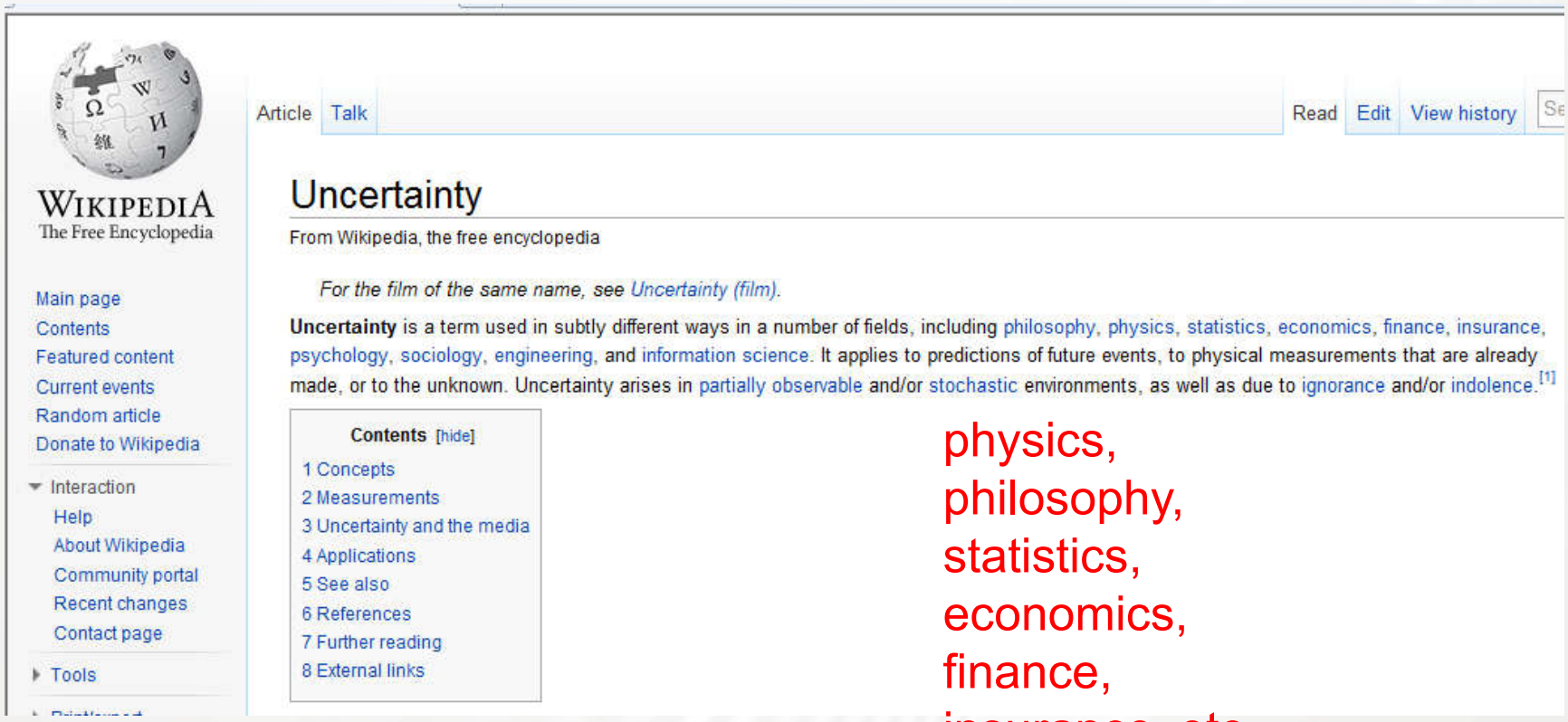
Still impressive !

What is Uncertainty ?

What does external world know about uncertainty ?



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Uncertainty

From Wikipedia, the free encyclopedia

For the film of the same name, see Uncertainty (film).

Uncertainty is a term used in subtly different ways in a number of fields, including philosophy, physics, statistics, economics, finance, insurance, psychology, sociology, engineering, and information science. It applies to predictions of future events, to physical measurements that are already made, or to the unknown. Uncertainty arises in partially observable and/or stochastic environments, as well as due to ignorance and/or indolence.[1]

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physics,
philosophy,
statistics,
economics,
finance,
insurance, etc.

Uncertainty is a term used in subtly different ways in a number of fields…

Introduction

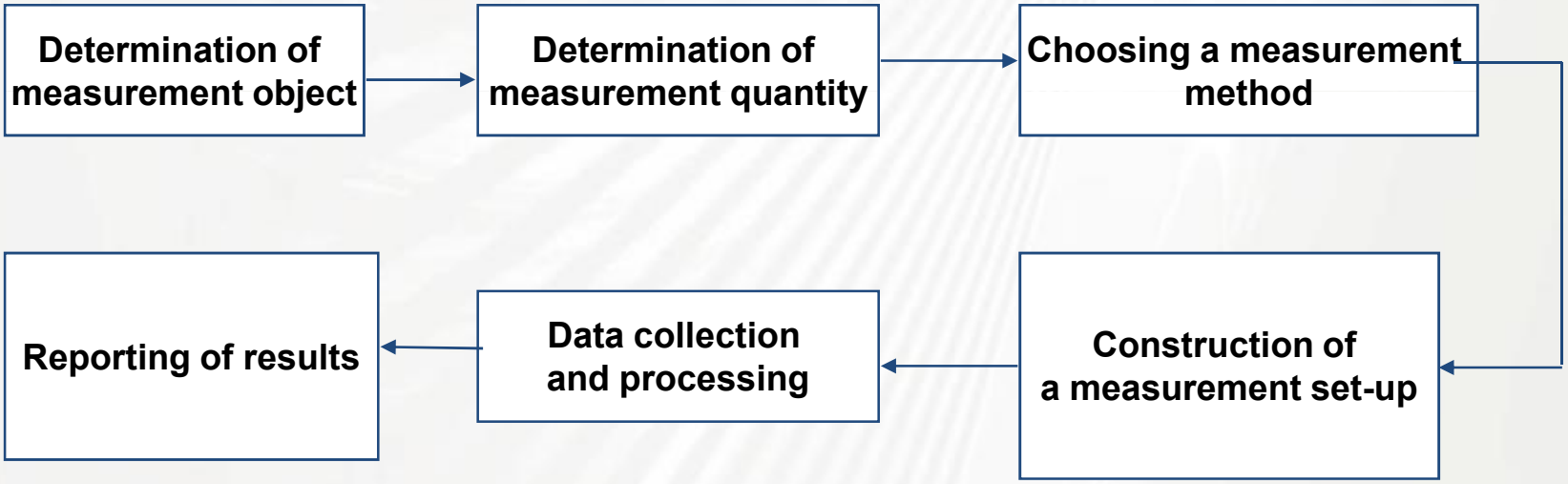
When reporting the result of a measurement of a physical quantity, it is obligatory to provide the **quantitative indication** of the quality of the result and its reliability.

The main indicator of the quality and reliability of the measurement result is **the uncertainty**.

Terms and Definitions

MEASUREMENT

Process of experimentally obtaining one or more quantity values that can be attributed to a quantity.



Measurement

- Measuring Equipment:
  - Technical Specification
  - Calibration
  - Traceability
- Environmental Conditions
- Method
- Staff

Terms and Definitions

Result of a measurement

Value attributed to a measurand, obtained by measurement.

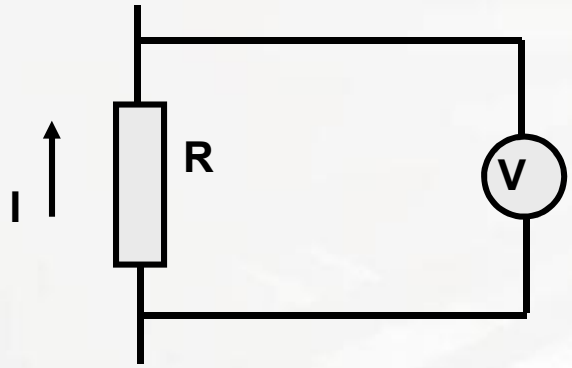
RESULT

The Indication

The Uncorrected Result

The Corrected Result

Example



QUANTITY	RESULT
Voltage, V	Indication
Current in accordance wth equation (1)	The Uncorrected Result
Current in accordance with equation (2)	The Corrected Result

Value indicated by Voltmeter: V

Resistance value from the certificate: Rs

According to the Ohm's Law:

$$I = \frac{V}{R} \qquad (1)$$

$$V = k \cdot V_i$$

$$R = R_s\left[1 + \alpha(t - t_0)\right]$$

$$I = \frac{k \cdot V_i}{R_s\left[1 + \alpha(t - t_0)\right]} \qquad (2)$$

Terms and Definitions

ACCURACY

Closeness of agreement between a measured quantity value and a true quantity value of the measurand.

The concept measurement accuracy is not given a numerical value, but a measurement is said to be accurate when it offers a smaller measurement uncertainty.

PRECISION

Closeness of agreement between indications obtained by replicate measurements on the same or similar objects under specified conditions.

Measurement precision is usually expressed numerically by measures of imprecision, such as standard deviation, variance, or coefficient of variation under the specified conditions of measurement.

Difference Between Accuracy and Precision

High accuracy,
Low precision

High accuracy,
High precision

Low accuracy,
High precision

Low accuracy,
Low precision

Terms and Definitions

EXPERIMENTAL STANDARD DEVIATION

Parameter, which characterize the dispersion of results of repeated measurements and calculated by the following formula:

$$s_{x_i} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\overline{x} - x_i)^2}$$

In this equation:

x_i : result of ith measurement,

$\overline{x}$: average of n measurement results

Terms and Definitions

REPEATABILITY

Closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurements.

Repeatability conditions include:

- the same measurement procedure
- the same observer
- the same measurement instrument under the same conditions
- the same location
- repetition over a short period of time

Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results.

Terms and Definitions

REPRODUCIBILITY

Closeness of the agreement between the results of successive measurements of the same measurand carried out under changed conditions of measurements.

The changed conditions may include:

- principle of measurement
- method of measurement
- observer
- measuring instrument
- reference standard
- location
- conditions of use
- time

A valid statement of reproducibility requires specification of the conditions changed.

Terms and Definitions

UNCERTAINTY

Non-negative parameter characterizing the **dispersion** of the quantity values being be attributed a measurand, based on the information used.

Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviation.

The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distribution based on experience or other information.

Terms and Definitions

UNCERTAINTY EVALUATION METHODS

Type A Evaluation of Standard Uncertainty:

Method of evaluation of uncertainty by the statistical analysis of series of observations.

Type B Evaluation of Standard Uncertainty:

Method of evaluation of uncertainty by means of other than the statistical analysis of series of observations.

Terms and Definitions

COMBINED STANDARD UNCERTAINTY

Standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model.

EXPANDED UNCERTAINTY

Product of a combined standard measurement uncertainty and a factor larger than the number one.

Sources of Uncertainty In Measurement

In practice, there are many possible sources of uncertainty:

- Incomplete definition of the measurand
- Imperfect realization of the definition of the measurand
- Effect of environmental conditions
- Personal bias in reading analogue instruments
- Finite instrument resolution or discrimination threshold
- Inexact values of measurement standards and reference materials
- Inexact values of constants and other parameters obtained from external sources
- Approximations and assumptions incorporated in the measurement method and procedure
- Random variations in repeated observations

Effect of Environmental Conditions

- Temperature
- Relative humidity
- Barometric pressure
- Electric or magnetic fields
- Gravity
- Electrical supplies to measuring equipment
- Air flow
- Vibration
- Light and optical reflections

Steps In Evaluation of Uncertainty

- Determination of mathematical model for the measurement (Realization of Model Function)
- Determination of uncertainty components
- Calculation of values of uncertainty components
- Evaluation of combined uncertainty
- Evaluation of expanded uncertainty
- Reporting of measurement results with uncertainty

Realization of Model Function

Model function defines the relationship between maesurement result (output quantity) and all quantities (input quantities) influencing and contributing to it.

$$y = F(x_1, x_2, x_3, \ldots, x_n)$$

$x_1, x_2, \ldots, x_n$: input quantities

y : output quantity (result)

Examples of Model Functions

Calibration of Mass Standard

$$m_x = m_S + \delta d_D + \delta m + \delta m_c + \delta B$$

Calibration of Hand-held Multimeter

$$E_x = V_{ix} - V_S + \delta V_{ix} - \delta V_S$$

Calibration of Gauge Block

$$L_x = L_S + L_D + \delta L - \left[L(\alpha\delta t + \delta\alpha\delta T)\right] + D_C + \delta C + L_{V(x)} + L_r$$

Reference: EA-4/02 and M3003 documents

Statistical Distributions

Main probability distributions used for an estimation of uncertainty components

Normal

Rectangular

Triangular

Trapezoidal

U-shape

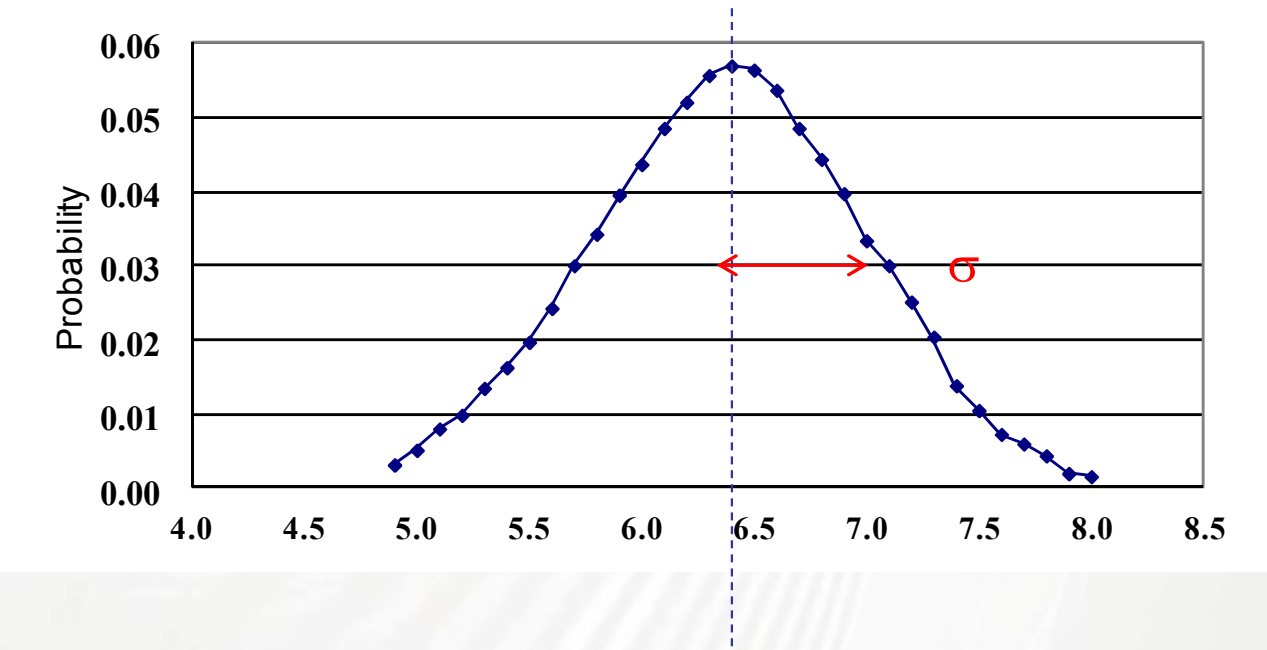
Normal Distribution

Result	Number (n)	p=n/N N=6000
4.9	22	0.003
5.0	36	0,.05
5.1	56	0.008
5.2	69	0.010
5.3	94	0.013
5.4	114	0.016
5.5	138	0.020
5.6	170	0.024
5.7	210	0.030
5.8	240	0.034
5.9	277	0.040

Result	Number (n)	p=n/N N=6000
6.0	306	0.044
6.1	340	0.049
6.2	365	0.052
6.3	390	0.056
6.4	399	0.057
6.5	395	0.056
6.6	376	0.054
6.7	340	0.049
6.8	311	0.044
6.9	278	0.040
7.0	230	0.033

Result	Number (n)	p=n/N N=6000
7.1	210	0.030
7.2	176	0.025
7.3	143	0.020
7.4	91	0.013
7.5	74	0.011
7.6	51	0.007
7.7	42	0.006
7.8	31	0.004
7.9	14	0.002
8.0	11	0.002
AVERAGE		6.45

Normal Distribution



Average of the Results

Graphical Representation of the Results

Normal Distribution

Normal Distribution = Gaussian Distribution

The distribution of the results within population is formulated by the equation:

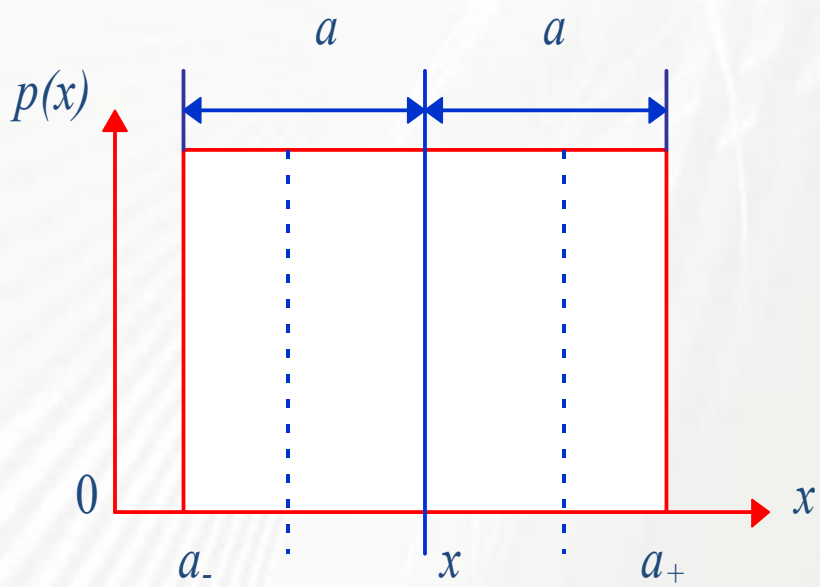
$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

$$z = \frac{x-\mu}{\sigma}; \qquad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{N}(x-\mu)^2}$$

x	Measurement results (Random Variable)
μ	True value of the measurand
σ	Standard deviation

Rectangular Distribution

The distribution is used when the only known information is that value of the quantity can be found with equal probability within defined interval.



$$u(x) = \frac{a_+ - a_-}{2\sqrt{3}}, \qquad u(x) = \frac{a}{\sqrt{3}}$$

Rectangular Distribution

WHY $\sqrt{3}$?

Probability Distribution Function $f(x) = \frac{1}{2a}$ $-a < x < a$

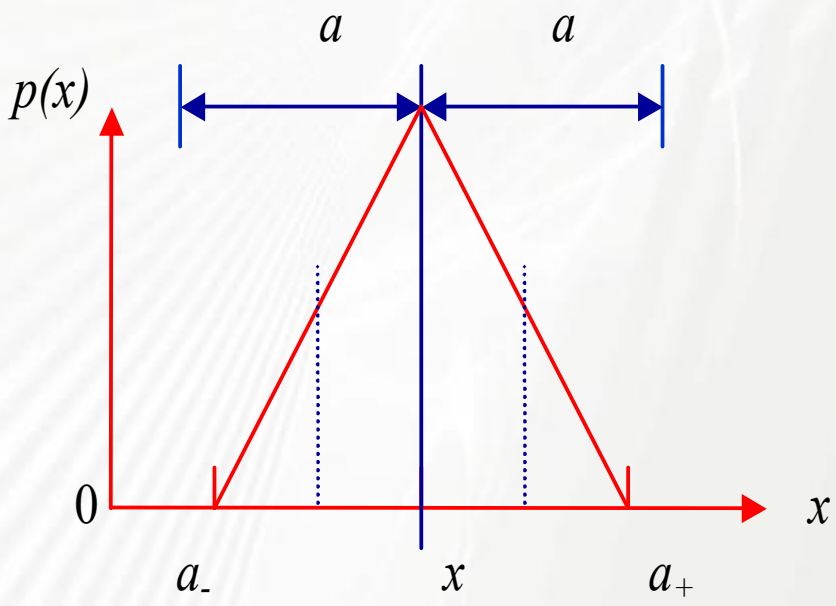
$$V(x) = \int x^2 f(x) dx$$

$$V(x) = \int_{-a}^{a} \frac{x^2}{2a} dx = \frac{1}{2a}\left(\frac{x^3}{3}\right)\Bigg|_{-a}^{+a} = \left(\frac{1}{6a}\right)\left(a^3 - (-a)^3\right) = \frac{2a^3}{6a} = \frac{a^2}{3}$$

$$u(x) = \sqrt{V(x)} = \frac{a}{\sqrt{3}}$$

Triangular Distribution

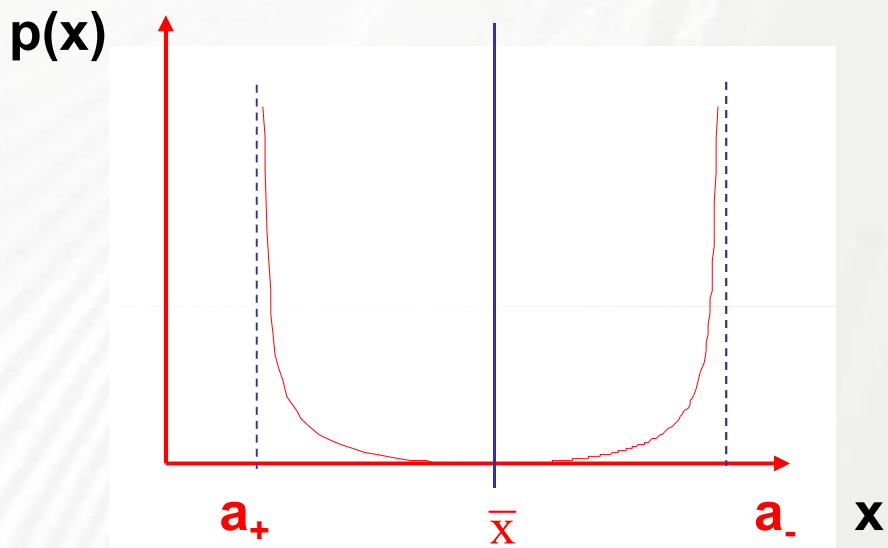
The distribution is used when the known information is that value of the quantity can be found within defined interval, but probability of finding the value in the center of the interval is higher.



$$u(x) = \frac{a_+ - a_-}{2\sqrt{6}}, \qquad u(x) = \frac{a}{\sqrt{6}}$$

U-Shape Distribution

The distribution is used when the known information is that value of the quantity can be found within defined interval, but probability of finding the value closer to boundaries of the interval is higher.



$$u(x) = \frac{a_+ - a_-}{2\sqrt{2}}, \qquad u(x) = \frac{a}{\sqrt{2}}$$

Type A Evaluation of Uncertainty

Evaluation of uncertainty by means of statistical analysis (A-type) is applied to the result of repeated measurements.

Statistical Distributions used in Type A Evaluation

Normal Distribution
Student Distribution

In case of only one measurement result
Type A evaluation is inapplicable !

Type B Evaluation of Uncertainty

Evaluation of uncertainty by means of other than statistical analysis is called Type B evaluation of uncertainty.

The information used for Type B Evaluation

- Previous measurement data
- Experience with or general knowledge of the behaviour and properties of relevant materials and instruments
- Manufacturer's specifications
- Data provided in calibration and other certificates
- Uncertainties assigned to reference data taken from handbooks.

Type B Evaluation of Uncertainty

Statistical Distributions used in Type B Evaluation

- Rectangular Distribution
- Triangular Distribution
- U-shape Distribution
- Trapezoidal Distribution

Calculation of Combined Uncertainty

The standard uncertainty of y, where y is the estimate of the measurand Y and thus the result of measurement, is obtained by appropriately combining the standard uncertainties of input estimates.

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i)}$$

In this equation, $c_i = \frac{\partial f}{\partial x_i}$ are sensitivity coefficients.

Note: The formula is valid for uncorrelated input quantities.

Combined Uncertainty

In case of correlated input quantities, the combined uncertainty is calculated by using following formula:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}} = \sum_{i=1}^{N}\left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j)$$

Combined Uncertainty

If the Model Function is presented as:

$$y = a_1x_1 + a_2x_2 + \ldots + a_nx_n = \sum_{i=1}^{n} a_ix_i$$

and a_i =Const, combined uncertainty is calculated by the following equation:

$$u_c(y) = \sqrt{\sum_{i=1}^{n} [a_i u(x_i)]^2}$$

In cases, when $a_i = \pm 1$,

$$u_c(y) = \sqrt{\sum_{i=1}^{n} u^2(x_i)} = \sqrt{u^2(x_1) + u^2(x_2) + \ldots + u^2(x_n)}$$

Combined Uncertainty

If the Model Function is presented as:

$$y = x_1^{p_1} x_2^{p_2} \ldots x_n^{p_n} = \prod_{i=1}^{n} x_i^{p_i}$$

and p_i =Const, combined uncertainty is calculated by the following equation:

$$\frac{u_c(y)}{y} = \sqrt{\sum_{i=1}^{n} \left[p_i \frac{u(x_i)}{x_i} \right]^2}$$

In cases, when $p_i = \pm 1$,

$$\frac{u_c(y)}{y} = \sqrt{\sum_{i=1}^{n} \left[\frac{u(x_i)}{x_i} \right]^2} = \sqrt{\left(\frac{u(x_1)}{x_1} \right)^2 + \left(\frac{u(x_2)}{x_2} \right)^2 + \ldots + \left(\frac{u(x_n)}{x_n} \right)^2}$$

Expanded Uncertainty

The expanded uncertainty is obtained by multiplying the combined standard uncertainty $u_c(y)$ by a coverage factor k.

$$U(y)=k\cdot u_c(y)$$

Coverage Factor and Confidence Level

Confidence Level p (%)	Coverage Factor k_p
68.27	1
90	1.645
95	1.960
95.45	2.0
99	2.576

Above data is valid for the case of normal distribution.

Reporting Uncertainty

The minimum requirements for the reporting uncertainty

- Describe clearly the methods used to calculate the measurement result and its uncertainty from the experimental observations and input data
- List all uncertainty components and document fully how they were evaluated
- Present the data analysis in such way that each of its important steps can be readily followed and the calculation of the reported result can be independently repeated if necessary
- Give all corrections and constants used in the analysis and their sources

Reporting Uncertainty

Other requirements

- Give a full description of how the measurand Y is defined
- Give the estimate y of the measurand Y and its combined standard uncertainty $u_c(y)$; the units of y and $u_c(y)$ should always be given
- Include the relative combined uncertainty $u_c(y)/|y|$, when appropriate

Reporting Uncertainty

Report measurement result in Calibration Certificates / Measurement Reports in the following way:

$$y \pm U$$

Here,

y : Average of the measurement results,

$U = k \cdot u_c$: Expanded uncertainty

Provide following explanatory notes:

The reported expanded uncertainty of measurement is stated as a standard uncertainty of measurement multiplied by the coverage factor k=2, which for a normal distribution corresponds to a coverage probability (confidence level) of approximately 95%. Expanded uncertainty is calculated in accordance with GUM and EA-4/02 documents.

Summary of Procedure for Uncertainty Evaluation

- Express mathematically relationship between the measurand y and the input quantities x_i

$$y = F(x_1, x_2, x_3, \ldots, x_n)$$

- Identify and apply all significant corrections
- List all sources of uncertainties, specify uncertainty components
- Calculate the standard uncertainties by Type B evaluation assuming the proper probability distributions for them

Summary of Procedure for Uncertainty Evaluation

For repeated series observation calculate experimental standard deviation

$$V(x) = s^2(x) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s(x) = \sqrt{V(x)}$$

Then standard deviation of the mean

$$u(\bar{x}) = s(\bar{x}) = \frac{s(x)}{\sqrt{n}}$$

Summary of Procedure for Uncertainty Evaluation

Calculate combined uncertainty

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i} \right]^2 u^2(x_i)}$$

Calculate expanded uncertainty

$$U(y) = k \cdot u_c(y)$$

Summary of Procedure for Uncertainty Evaluation

Report the result of measurement comprising the estimate y of the measurand, the associated expanded uncertainty U, coverage factor and confidence level.

y=Estimate (e.g. average) ± Expanded Uncertainty

Provide the neccessary explanatory notes.

References

- Guide to the Expression of Uncertainty in Measurement, ISO 1995
- Expression of the Uncertainty of Measurements in Calibration, EA-4/02, European Accreditation, 1999
- The Expression of Uncertainty and Confidence in Measurements, UKAS Publication M3003, UKAS, UK, 1997
- Quantifying Uncertainty in Analytical Measurements, URACHEM/CITAC/CG 4 Guide, 2001
- T.G.Beckwith, R.D.Marangoni, J.H.Lienhard, Mechanical Measurements, 5th edition,1993
- C.F.Dietrich Uncertainty, Calibration and Probability, Adam Hilger, 1991
- B.N.Taylor and C.E.Kuyatt, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST Tech. Note 1297, 1993
- R.P.Benedict Fundamentals of Temperature, Pressure and Flow Measurements, John Wiley & Sons Inc.,1984

Useful Links

- http://physics.nist.gov/cuu/Uncertainty/index.html
- http://www.measurementuncertainty.org
- http://www.ukas.com/information_centre/technical/technical_uncertain.asp
- http://www.uncertaintymanager.com
- http://www.european-accreditation.org
- http://www.npl.co.uk/publications/good_practice/uncert
- http://www.ilac.org/downloads/Ilac-g17.pdf
- http://www.metrodata.de/papers/resistor_en.pdf

EMPIR EURAMET

The EMPIR initiative is co-funded by the European Union's Horizon 2020 research and innovation programme and the EMPIR Participating States

RFMICROWAVE

15RPT01 RFMicrowave

Development of RF and Microwave Metrology Capability

Thanks for attention!