Uncertainty Evaluation of A Power Sensor Calibration

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Why do we need uncertainty?

When reporting the result of a measurement of a physical quantity, it is obligatory to provide the quantitative indication of the quality of the result and its reliability.

The main indicator of the quality and reliability of the measurement result is the uncertainty.
Uncertainty evaluation

- Generally 2 methods are used in metrology

    - GUM is based on the law of propagation of uncertainty

  - **Monte Carlo** method based on document *Evaluation of measurement dat – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method*, 2008
    - MCM is based on the propagation of distribution

documents online

Uncertainty evaluation

- The *standard uncertainty* of the result of a measurement is obtained from the values of a number of other quantities. It is called also combined standard uncertainty and denoted by $u_c$.

- It is the estimated standard deviation associated with the result and is equal to the positive square root of the *combined variance* obtained from all variance and covariance components.

- The *combined standard uncertainty* $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$, which is given by

\[
  u_c^2(y) = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)
\]

Where,

\[ Y = f(X_1, X_2, ..., X_N) \]

Each $u(x)$ is a standard uncertainty (Type A or Type B evaluation).
Uncertainty evaluation

Type A Evaluation of Standard Uncertainty:

Method of evaluation of uncertainty by the statistical analysis of series of observations.

Experimental (type A) standard uncertainty of the mean is

\[ u(s) = \frac{1}{\sqrt{n}} \cdot \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2} \]

\( n \): number of observation
\( X_i \): \( n^{th} \) observed value
\( \bar{X} \): Average of the \( n \) observation
Type B Evaluation of Standard Uncertainty:

Method of evaluation of uncertainty by means of other than the statistical analysis of series of observations. Type B standard uncertainty may include,

- previous measurement data
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments
- manufacturer's specifications
- Data provided in calibration and other certificates
- uncertainties assigned to reference data taken from handbooks
Uncertainty evaluation

- Other parts \((\delta f / \delta X_i)\) of the uncertainty are based on a first-order Taylor series approximation of the function \(Y = f(X_1, X_2, \ldots, X_n)\).

- These derivatives, often called *sensitivity coefficients*, describe how the output estimate \(y\) varies with changes in the values of the input estimates.

\[
 u_c^2(y) = \sum_{i=1}^{N} \left[ c_i u(x_i) \right]^2 \equiv \sum_{i=1}^{N} u_i^2(y)
\]

where

\[
 c_i \equiv \frac{\partial f}{\partial x_i}, \quad u_i(y) \equiv |c_i| u(x_i)
\]
Uncertainty evaluation

- Effective degrees of freedom:
  if the probability distribution of \( y \) is not known exactly, central limit theorem can be used. It assumes that \( \frac{(y - Y)}{u(y)} \) has the Student’s \( t \)-distribution with effective degrees of freedom \( v_{\text{eff}} \).

\[
v_{\text{eff}} = \frac{\left(\frac{u_c(y)}{N}\right)^4}{\sum_{i=1}^{N} \left(\frac{u_i(y)}{v_i}\right)^4}
\]

\( v_{\text{eff}} \) obtained from the Welch-Satterthwaite equation.

- Expended uncertainty

\[
U(y) = t_p u(y) = ku(y)
\]
Uncertainty evaluation

- variances of normal and rectangular distributions

<table>
<thead>
<tr>
<th>probability distribution</th>
<th>( z_{\max} )</th>
<th>( k )</th>
<th>probability distribution</th>
<th>( z_{\max} )</th>
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<td>normal (Gauss)</td>
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<td>uniform - rectangular</td>
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\[ a \] \( \sqrt{3} \)
\[ 1,73 \]
– If the inputs of $Y$ are correlated

$$u_c^2(y) = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$
Uncertainty evaluation

- **GUF method scheme**

  \[
  Y = f(X_1, \ldots, X_N)
  \]
  
  estimates \( x_1, \ldots, x_N \) of \( X_1, \ldots, X_N \)

  \[
  y = f(x_1, \ldots, x_N)
  \]
  
  estimate \( y = f(x_1, \ldots, x_N) \) of \( Y \)

  \[
  u(y) = \sum_{i=1}^{N} (c_i u(x_i))^2
  \]
  
  standard uncertainty \( u(y) \)

  \[
  \delta f / \delta x_N
  \]
  
  sensitivity coefficients \( c_1, \ldots, c_N \)

  \[
  k u(y)
  \]
  
  expanded uncertainty \( U \)

  \[
  y \pm U
  \]
  
  coverage interval \( y \pm U \) for \( Y \)
Uncertainty evaluation for calibration factor (CF) measurement
Uncertainty evaluation for CF measurement

- Simple CF measurement;

Mismatch ratio $M=1$, i.e. Standard and DUT power sensor mismatch are assumed to be equal.

$$CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC,std}} M$$

$$M = \frac{M_{DUT}}{M_{std}} = \frac{1 - \Gamma_{DUT} \Gamma_G}{1 - \Gamma_{std} \Gamma_G}$$

Diagram:

- Signal generator
- Reference power standard
- Power sensor to be calibrated

EMPIR 15RPT01 workshop, 7.11.2016, METAS
Uncertainty evaluation for CF measurement

- Simple CF measurement:

\[
CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC,std}} M
\]

Mismatch uncertainty,

\[
u^2(M) = 2|\Gamma_G||\Gamma_{std}| + 2|\Gamma_G||\Gamma_{DUT}|
\]

U-distribution

\begin{figure}
\centering
\includegraphics[width=\textwidth]{diagram.png}
\end{figure}
Uncertainty evaluation

- Simple CF measurement – **mismatch correction**
  - complex reflection coefficients considered
  - real + imaginary is better for numerical calculations compared to the magnitude + phase
  
  \[
  CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{|1 - \Gamma_{DUT} \Gamma_G|^2}{|1 - \Gamma_{std} \Gamma_G|^2}
  \]

  \[
  \begin{align*}
  a &= \text{Re}\{\Gamma_{std}\} & d &= \text{Im}\{\Gamma_{DUT}\} \\
  b &= \text{Im}\{\Gamma_{std}\} & e &= \text{Re}\{\Gamma_G\} \\
  c &= \text{Re}\{\Gamma_{DUT}\} & f &= \text{Im}\{\Gamma_G\}
  \end{align*}
  \]

  \[
  CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{1 + 2df - 2ce + c^2e^2 + d^2e^2 + c^2f^2 + d^2f}{1 + 2bf - 2ae + a^2e^2 + b^2e^2 + a^2f^2 + b^2f^2}
  \]
Uncertainty evaluation

- Simple CF measurement – **mismatch correction**
  - sensitivity coefficients:

\[
\frac{\delta CF_{DUT}}{\delta CF_{std}}, \quad \frac{\delta CF_{DUT}}{\delta P_{DC,std}}, \quad \frac{\delta CF_{DUT}}{\delta P_{DC,DUT}}
\]

\[
\frac{\delta CF_{DUT}}{\delta a}, \quad \frac{\delta CF_{DUT}}{\delta Pb}, \quad \ldots, \quad \frac{\delta CF_{DUT}}{\delta f}
\]

- normal probability distribution is assumed for real and imaginary parts of the reflection coefficients
Uncertainty evaluation

- **CF measurement**—power splitter and mismatch correction

\[
CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC,\text{std}}} \frac{P_{3\text{std}}|1-\Gamma_{DUT}\Gamma_G|^2}{P_{3\text{DUT}}|1-\Gamma_{\text{std}}\Gamma_G|^2}
\]

Diagram:
- **Signal generator**
- **Monitoring power sensor + meter**
  - Level control
- **Reference power standard**
- **Power sensor to be calibrated**
- \(Z_0\)
- \(\Gamma_{\text{std}}\)
- \(\Gamma_{EG}\)
- \(\Gamma_{DUT}\)

- \(P_{DC,\text{std}}\) when ref. sensor connected
- \(P_{3\text{std}}\) when ref. sensor connected
- \(P_{3\text{DUT}}\) when DUT sensor connected
Uncertainty evaluation

\[ CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{P_{3, std}}{P_{3, DUT}} \left| 1 - \Gamma_{DUT} \Gamma_G \right|^2 \]

\[ \Gamma_{DUT} : \text{Reflection coefficient of the DUT sensor} \]
\[ \Gamma_{std} : \text{Reflection coefficient of the STD sensor} \]
\[ \Gamma_G : \text{Reflection coefficient of the DUT and STD sensor connection port} \]

\[ \text{CF}_{\text{std}} : \text{Calibration factor of the standard power sensor/thermistor mount} \]
\[ P_{D,\text{DC, DUT}} : \text{Measured power from the power meter (PM) connected to the DUT sensor} \]
\[ P_{3,\text{DUT}} : \text{Measured power from the monitor PM when DUT sensor measured} \]
\[ P_{D,\text{DC, std}} : \text{Measured power from the PM connected to the STD sensor} \]
\[ P_{3,\text{std}} : \text{Measured power from the monitor PM when STD sensor measured} \]
Uncertainty evaluation

\[ u(CF_{std}) : \text{Uncertainty of the calibration factor of the standard power sensor/thermistor mount} \]

\[ u(P_{DC,DUT}) : \text{Uncertainty of the measured power from the power meter (PM) connected to the DUT sensor} \]

\[ u(P_{3DUT}) : \text{Uncertainty of the measured power from the monitor PM when DUT sensor measured} \]

\[ u(P_{DC, std}) : \text{Uncertainty of the measured power from the PM connected to the STD sensor} \]

\[ u(P_{3std}) : \text{Uncertainty of the measured power from the monitor PM when STD sensor measured} \]

\[ u(\Gamma_{DUT}) : \text{Uncertainty of the reflection coefficient of the DUT sensor} \]

\[ u(\Gamma_{std}) : \text{Uncertainty of the reflection coefficient of the STD sensor} \]

\[ u(\Gamma_{G}) : \text{Uncertainty of the reflection coefficient of the DUT and STD sensor connection port} \]
Uncertainty evaluation

\[ CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC,STD}} \frac{P_{3std}}{P_{3DUT}} \frac{M_D}{M_s} \]

\[ MM = \frac{M_D}{M_s} = \frac{1 - \Gamma_{DUT} \Gamma_G}{1 - \Gamma_{std} \Gamma_G} \]

\[ c_{CFSTD} = \frac{\partial CF_{DUT}}{\partial CF_{STD}} = \frac{CF_{DUT}}{CF_{STD}} \]
\[ c_{PDC,DUT} = \frac{\partial CF_{DUT}}{\partial P_{DC,DUT}} = \frac{CF_{DUT}}{P_{DC,DUT}} \]
\[ c_{PDC,STD} = \frac{\partial CF_{DUT}}{\partial P_{DC,STD}} = -\frac{CF_{DUT}}{P_{DC,STD}} \]
\[ c_{P3DUT} = \frac{\partial CF_{DUT}}{\partial P_{3DUT}} = -\frac{CF_{DUT}}{P_{3DUT}} \]
\[ c_{P3std} = \frac{\partial CF_{DUT}}{\partial P_{3std}} = \frac{CF_{DUT}}{P_{3std}} \]
\[ c_{MM} = \frac{\partial CF_{DUT}}{\partial MM} = \frac{CF_{DUT}}{MM} \]
Uncertainty evaluation

\[ CF_{\text{DUT}} = CF_{\text{std}} \frac{P_{\text{DC,DUT}}}{P_{\text{DC, std}}} \frac{P_{3\text{std}}}{P_{3\text{DUT}}} \ MM \]

\[ u^2(CF_{\text{DUT}}) = c_{CF,\text{std}}^2 u^2(CF_{\text{std}}) + c_{P_{DC,\text{DUT}}}^2 u^2(P_{DC,\text{DUT}}) + c_{P_{DC,\text{std}}}^2 u^2(P_{DC,\text{std}}) + c_{P_{3\text{DUT}}}^2 u^2(P_{3\text{DUT}}) + c_{MM}^2 u^2(MM) \]
Uncertainty evaluation

\[ CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{P_{3DUT}}{P_{3DUT}} MM \]

If \( v_{eff} \) is greater than 50, \( t_p (=k) \) can be assumed 2 for %95.45.
Uncertainty evaluation

\[ CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{P_{3std}}{P_{3DUT}} \times MM \]

\[ U(CF_{DUT}) = k u(CF_{DUT}) \]
Uncertainty evaluation

\[
CF_{DUT} = CF_{std} \frac{P_{DC,DUT}}{P_{DC,std}} \frac{P_{3_{std}}}{P_{3_{DUT}}} \times MM
\]

\[
\begin{align*}
DUT & = \frac{\partial f}{\partial x_i} \\
& = \frac{\partial f}{\partial y} \\
& = \sum_{i=1}^{N} (c_i n(x_i))
\end{align*}
\]

\[
U(y) = k u(y)
\]


covariance interval \( y \pm U \) for \( Y \)
Mathematical model

- calibration with an adaptor/attenuator

\[ \text{CF}_{DUT} = \text{CF}_{std} \frac{P_{DC,DUT}}{P_{DC, std}} \frac{P_{3, std}}{P_{3, DUT}} \frac{1}{|S_{21A}|^2} \left| \frac{1 - \Gamma_{DUT} S_{22A} - \Gamma_G \Gamma_{A-DUT}}{1 - \Gamma_{std} \Gamma_G} \right|^2 \]

\[ \Gamma_{A-DUT} = S_{11A} + \Gamma_{DUT} S_{21A} S_{12A} - \Gamma_{DUT} S_{11A} S_{22A} \]
Mathematical model

- calibration with an **attenuator**

\[
CF_{DUT} = \frac{\frac{\frac{P_{DC,DUT}}{P_{DC,\text{std}}}}{P_{3\text{std}}}}{P_{3\text{DUT}} \left| S_{21A} \right|^2} \left( \frac{1 - \Gamma_{DUT} S_{22A} - \Gamma_G \Gamma_{A-DUT}}{1 - \Gamma_{\text{std}} \Gamma_G} \right)^2
\]

Effect of transmission coefficient of the attenuator

\[
c_{s_{21A}} = \frac{\partial CF_{DUT}}{\partial S_{21A}}
\]

Uncertainty of this part is already discussed

- Uncertainty of forward transmission coefficient of the attenuator: \(u(S_{21A})\)
- Uncertainty of reverse transmission coefficient of the attenuator: \(u(S_{12A})\)
- Uncertainty of Port 1 reflection coefficient of the attenuator: \(u(S_{11A})\)
- Uncertainty of Port 2 reflection coefficient of the attenuator: \(u(S_{22A})\)

\[
c_{MM} = \frac{\partial CF_{DUT}}{\partial MM}
\]
Connector effect

- Effect of asymmetry of the connector should be included to the CF measurements

![Diagram showing three orientations at 120° intervals]

$$\overline{CF_{DUT}} = \frac{1}{N} \sum_{i=1}^{N} CF_{DUT_i}$$

$$u_{conn}^2 = \frac{1}{N(N-1)} \sum_{i=1}^{N} (CF_{DUT_i} - \overline{CF_{DUT}})^2$$
EMPIR project “15RPT01 Development of RF and microwave metrology capability”
http://rfmw.cmi.cz/

Thank you for attention
Monte Carlo scheme

1. Measure model: $Y = f(X_1, \ldots, X_N)$
2. Probability distribution for $X_i$
3. $M$ samples of the MC method
4. Coverage probability $p$

- $M$ sets of values of $X_1, \ldots, X_N$ with corresponding probability distribution
- $M$ values of measured quantity corresponding to the value sets
- Sorted values of the function of the measured quantity: discrete representation of the distribution function for $Y$

Estimate $y$ for $Y$ and associated standard uncertainty $u(y)$

Coverage interval for $Y$